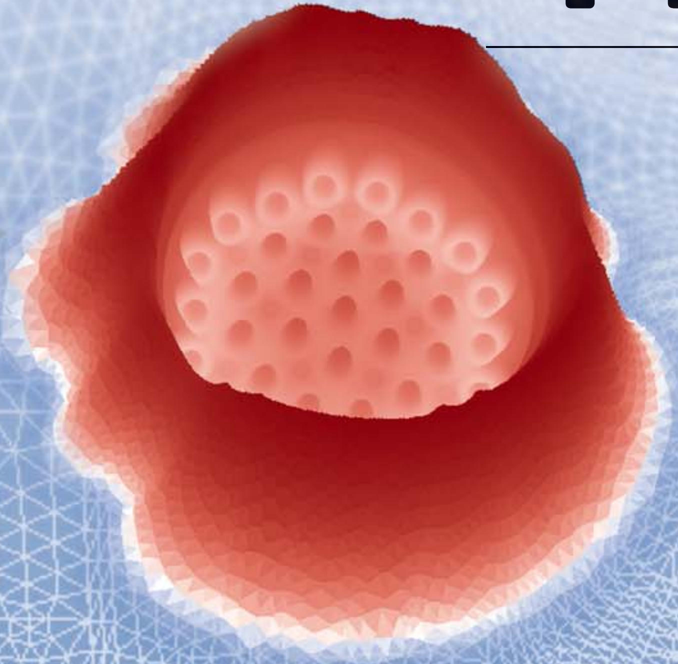
NUCLEAR ENGINEERING

Paul Reuss

# NEUTRON PHYSICS



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# Neutron Physics

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NUCLEAR ENGINEERING

# Neutron Physics

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The author would like to thank Nova Traduction (K. Foster) and Chris Latham for the translation of his book.

Cover illustrations: Jules Horowitz (1921-1995), a highly talented physicist, founded the French school of neutron physics. In 2014, the Jules Horowitz reactor being built at Cadarache will become the main irradiation reactor in the world (100 MWth) for research on materials and nuclear fuels.
*In the background, the meshing for a neutron physics core calculation and in the foreground the power distribution, result of this calculation. (Documents courtesy of CEA.)*

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# Introduction to the *Nuclear Engineering* Collection

Within the French Atomic Energy Commission (CEA), the National Institute of Nuclear Science and Technology (INSTN) is a higher education institution operating under the joint supervision of the Ministries of Education and Industry. The purpose of the INSTN is to contribute to disseminating the CEA's expertise through specialised courses and continuing education, not only on a national scale, but across Europe and worldwide.

This mission is focused on nuclear science and technology, and one of its main features is a Nuclear Engineering diploma. Bolstered by the CEA's efforts to build partnerships with universities and engineering schools, the INSTN has developed links with other higher education institutions, leading to the organisation of more than twenty five jointly-sponsored Masters graduate diplomas. There are also courses covering disciplines in the health sector: nuclear medicine, radiopharmacy, and training for hospital physicists.

Continuous education is another important part of the INSTN's activities that relies on the expertise developed within the CEA and by its partners in industry.

The Nuclear Engineering course (known as 'GA', an abbreviation of its French name) was first taught in 1954 at the CEA Saclay site, where the first experimental piles were built. It has also been taught since 1976 at Cadarache, where fast neutron reactors were developed. GA has been taught since 1958 at the School for the Military Applications of Atomic Energy (EAMEA), under the responsibility of the INSTN. Since its creation, the INSTN has awarded diplomas to over 4400 engineers who now work in major companies or public-sector bodies in the French nuclear industry: CEA, EDF (the French electricity board), AREVA, Cogema, Marine Nationale (the French navy), IRSN (French TSO)... Many foreign students from a variety of countries have also studied for this diploma.

There are two categories of student: civilian and military. Civilian students will obtain jobs in the design or operation of nuclear reactors for power plants or research establishments, or in fuel processing facilities. They can aim to become expert consultants, analysing nuclear risks or assessing environmental impact. The EAMEA provides education for certain officers assigned to French nuclear submarines or the aircraft carrier.

The teaching faculty comprises CEA research scientists, experts from the Nuclear Safety and Radiation Protection Institute (IRSN), and engineers working in industry (EDF, AREVA, etc.). The main subjects are: nuclear physics and neutron physics, thermal hydraulics, nuclear materials, mechanics, radiological protection, nuclear instrumentation, operation and safety of Pressurised Water Reactors (PWR), nuclear reactor systems, and the nuclear fuel cycle. These courses are taught over a six-month period, followed by a final project that rounds out the student's training by applying it to an actual industrial situation.

These projects take place in the CEA's research centres, companies in the nuclear industry (EDF, AREVA, etc.), and even abroad (USA, Canada, United Kingdom, etc.). A key feature of this programme is the emphasis on practical work carried out using the INSTN facilities (ISIS training reactor, PWR simulators, radiochemistry laboratories, etc.).

Even now that the nuclear industry has reached full maturity, the Nuclear Engineering diploma is still unique in the French educational system, and affirms its mission: to train engineers who will have an in-depth, global vision of the science and the techniques applied in each phase of the life of nuclear installations from their design and construction to their operation and, finally, their dismantling.

The INSTN has committed itself to publishing all the course materials in a collection of books that will become valuable tools for students, and to publicise the contents of its courses in French and other European higher education institutions. These books are published by EDP Sciences, an expert in the promotion of scientific knowledge, and are also intended to be useful beyond the academic context as essential references for engineers and technicians in the industrial sector.

The European Nuclear Education Network (ENEN) fully supported INSTN, one of it founder members, in publishing this book. For ENEN this book constitutes the first of a series of textbooks intended for students and young professionals in Europe and worldwide, contributing to the creation of the European Educational Area.

Joseph Safieh
Nuclear Engineering Course Director
ENEN President

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**Table C.3.**

| Abscissa number | Weight |
|---|---|
| 0 | 1/3 |
| 1 | 4/3 |
| 2 | 1/3 |

- *Modified Simpson's rule:* let us take $N = 4$, i.e. $I$ a multiple of 4. We can then write the trapezoidal rule for the elementary steps $h$, $2h$ and $4h$. By combining the three formulae with suitable coefficients, we can make not only the $h^2$ terms vanish, but also the $h^4$ terms, thus obtaining a more precise formula which, in this case, has *error of order* $h^6$. The coefficients are as follows:

**Table C.4.**

| Abscissa number | |
|---|---|
| 0 | 14/45 |
| 1 | 64/45 |
| 2 | 24/65 |
| 3 | 64/45 |
| 4 | 14/45 |

- *Weddle's rule:* let us take $N = 6$, i.e. $I$ a multiple of 6. We can now combine formulae for the elementary step $h$, for the step $2h$ and for the step $3h$ to make the $h^2$ and $h^4$ terms vanish in the expression for the remainder. We thus obtain Weddle's rule, with *error of order* $h^6$. The coefficients are remarkably simple (especially if 3/10 is added as a factor): see Table C.5.

- *Modified Weddle's rule:* still using $N = 6$, i.e. $I$ a multiple of 6, we can combine not only formulae for the steps $h$, $2h$ and $3h$, but also the formula with step $6h$: we thus improve the formula, because the *error is then of order* $h^8$, but the coefficients are not as simple:

**Table C.5.**

| Abscissa | Weight of number Weddle's rule: | |
|---|---|---|
| | standard | modified |
| 0 | 3/10 | 41/140 |
| 1 | 15/10 | 216/140 |
| 2 | 3/10 | 27/140 |
| 3 | 18/10 | 272/140 |
| 4 | 3/10 | 27/140 |
| 5 | 15/10 | 216/140 |
| 6 | 3/10 | 41/140 |

- *Other elementary formulae*: this type of reasoning can obviously be pursued: here are the coefficients of three 12-step formulae, whose *errors* are of order $h^8$, $h^{10}$ and $h^{12}$ respectively (see Table C.6).

**Table C.6.**

| Abscissa number | Weight of 12-interval formulae | | |
|---|---|---|---|
| 0 | 10/35 | 49/175 | 41833/150150 |
| 1 | 56/35 | 288/175 | 248832/150150 |
| 2 | 0 | –27/175 | –29160/150150 |
| 3 | 80/35 | 448/175 | 395264/150150 |
| 4 | –4/35 | –63/175 | –63909/150150 |
| 5 | 56/35 | 288/175 | 248832/150150 |
| 6 | 24/35 | 134/175 | 118416/150150 |
| 7 | 56/35 | 288/175 | 248832/150150 |
| 8 | –4/35 | –63/175 | –63909/150150 |
| 9 | 80/35 | 448/175 | 395264/150150 |
| 10 | 0 | –27/175 | –29160/150150 |
| 11 | 56/35 | 288/175 | 248832/150150 |
| 12 | 10/35 | 49/175 | 41833/150150 |

### b) Newton-Cotes formulae

The Newton-Cotes formulae are obtained by systematically seeking the "best choice" of weights when $N$ has been chosen. With $N$ unknowns $w_i$ to be determined, we can write $N$ equations by writing out the precision for the successive mononomials up to $x^{N-1}$. We thus write out exact formulae to the order $N - 1$, with error of order $h^{N+1}$ if $N$ is even. We thus find, respectively, the trapezoidal rule, Simpson's rule, the modified Simpson's rule, and the modified Weddle's rule, for the values 1, 2, 4 and 6 of $N$. These formulae therefore turn out to be the best possible constant-step formulae for these values of $N$. For even values of $N$ beyond 6, we can find formulae of order $h^{N+2}$ higher than that of the elementary formulae. The odd values of $N$ are less interesting because, for reasons of symmetry, the formulae with $2n$ steps and $2n + 1$ steps are of the same order.

## C.11.3. Gauss formulae

As we can see from the tables (tables C.2 to C.6), the weights $w_i$ are increasingly dispersed in orders of magnitude as $N$ increases. There is one disadvantage to this *dispersion*: it *increases the sensitivity to numerical errors*. The advantage of moving by one order of precision thus ends up being counterbalanced by the increase in numerical errors, and so it is pointless to try to develop formulae with even larger values of $N$.

This observation led Chebyshev to construct the least sensitive formulae possible — i.e. *with weights w that are all equal* — and the most exact formulae possible, by adjusting the choice of abscissas $x_i$. This route, however, soon leads to a dead end, because beyond $N = 8$, the polynomial equation giving the values for $x_i$ has complex roots.

In practice, the Chebyshev formulae are of little benefit here, because the Gauss formulae are almost as insensitive to numerical errors as the Chebyshev formulae, but have a higher order of precision.